

## Lecture 15 - Oct. 31

### Bridge Controller

*Revising M0: Adding Event Guards  
Re-Generating/Re-Proving PO Sequents*

## Announcements/Reminders

- **Lab4** due tomorrow at noon
- **ProgTest2** next Wednesday, November 6

\*  $\forall n \leq d \vdash n-1 < d$  DEC

# Discharging POs of original m0: Invariant Preservation

**ML\_out/inv0\_1/INV**

$d \in \mathbb{N}$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $\vdash$   
 $n+1 \in \mathbb{N}$

x P2  $\therefore$  too many irrelevant hypotheses.

MON  $\frac{n \in \mathbb{N}}{\vdash n+1 \in \mathbb{N}}$  P2

**ML\_in/inv0\_1/INV**

$d \in \mathbb{N}$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $\vdash$  True  
 $n-1 \in \mathbb{N}$

MON  $\frac{n \in \mathbb{N}}{\vdash n-1 \in \mathbb{N}}$  ML- $\tau_1$

$n \in \mathbb{N}$   
 $\vdash$  ?  
 $n-1 \in \mathbb{N}$

new guard for ML- $\tau_1$   
 $n > 0$

unprovable

One possible resolution:

$\frac{H \vdash P}{H \vdash P \vee Q}$  OR.R1

$\frac{H1 \vdash G}{H1, H2 \vdash G}$  MON

$\frac{}{n \leq m + n - 1 < m}$  DEC

$\frac{}{n \in \mathbb{N} \vdash n+1 \in \mathbb{N}}$  P2

**ML\_out/inv0\_2/INV**

$d \in \mathbb{N}$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $\vdash$  True  
 $n+1 \leq d$

MON

$\frac{n \leq d}{\vdash n+1 \leq d}$

new guard not for ML-out:  $n \leq d$   
 not provable  $\because n$  might be equal to  $d$ .

**ML\_in/inv0\_2/INV**

$d \in \mathbb{N}$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $\vdash$   
 $n-1 \leq d$

DEC x  
 ARI  $\frac{P \leq 8}{P < 8} \frac{P = 8}{P = 8}$

$\frac{d \in \mathbb{N} \quad n \in \mathbb{N} \quad n \leq d}{\vdash n-1 < d}$   
 $\forall n-1 = d$

add some guard condition of ML- $\tau_1$ , so that the extra hypothesis can make this seg. provable.

OR-R1

$\frac{d \in \mathbb{N} \quad n \in \mathbb{N} \quad \vdash n-1 < d}{n \leq d}$

MON x

# PO/VC Rule of Invariant Preservation: Revised M0

constants: $d$	variables: $n$	ML_out <i>when <math>n &lt; d</math></i> begin $n := n + 1$ end
axioms: axm0_1 : $d \in \mathbb{N}$	invariants: inv0_1 : $n \in \mathbb{N}$ inv0_2 : $n \leq d$	ML_in <i>when <math>n &gt; 0</math></i> begin $n := n - 1$ end

$$\begin{array}{l} A(c) \\ I(c, v) \\ \boxed{G(c, v)} \\ \vdash \\ \boxed{I_i(c, E(c, v))} \end{array}$$

Q. How many PO/VC rules for model m0? 4

# Discharging **PO**s of revised m0: Invariant Preservation

**ML\_out/inv0\_1/INV**

Ex.

$d \in \mathbb{N}$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $n < d$   
 $\vdash$   
 $n + 1 \in \mathbb{N}$

**ML\_in/inv0\_1/INV**

$d \in \mathbb{N}$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $n > 0$   
 $\vdash$   
 $n - 1 \in \mathbb{N}$

new guard for ML\_in MON

$n > 0$   
 $\vdash$   
 $n - 1 \in \mathbb{N}$

P2

**ML\_out/inv0\_2/INV**

$d \in \mathbb{N}$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $n < d$   
 $\vdash$   
 $n + 1 \leq d$

new guard for ML\_out MON

$n < d$   
 $\vdash$   
 $n + 1 \leq d$

INC

**ML\_in/inv0\_2/INV**

Ex.

$d \in \mathbb{N}$   
 $n \in \mathbb{N}$   
 $n \leq d$   
 $n > 0$   
 $\vdash$   
 $n - 1 \leq d$

$$\frac{H \vdash P}{H \vdash P \vee Q} \text{ OR\_R1}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G} \text{ MON}$$

$$\frac{}{n \leq m \vdash n - 1 < m} \text{ DEC}$$

$$n < m \vdash n + 1 \leq m \text{ INC}$$

$$\frac{}{n \in \mathbb{N} \vdash n + 1 \in \mathbb{N}} \text{ P2}$$

$$0 < n \vdash n - 1 \in \mathbb{N} \text{ P2'}$$

# Discharging PO of **DLF**: Second Attempt

$$\frac{}{H, P \vdash P} \text{ HYP}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G} \text{ MON}$$

$$\frac{H, P \vdash R \quad H, Q \vdash R}{H, P \vee Q \vdash R} \text{ OR\_L}$$

$$\frac{H \vdash P}{H \vdash P \vee Q} \text{ OR\_R1}$$

$$\frac{H \vdash Q}{H \vdash P \vee Q} \text{ OR\_R2}$$

$$\begin{array}{l} d \in \mathbb{N} \\ d > 0 \\ n \in \mathbb{N} \\ \boxed{n \leq d} \quad n < d \vee n = d \\ \vdash \\ n < d \vee n > 0 \end{array} \text{ ARI}$$

$$\begin{array}{l} d \in \mathbb{N} \\ d > 0 \\ n \in \mathbb{N} \\ \boxed{n < d \vee n = d} \\ \vdash \\ n < d \vee n > 0 \end{array} \text{ MON}$$

$$\begin{array}{l} n < d \vee n = d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

$$\begin{array}{l} \text{OR\_L} \\ \left. \begin{array}{l} n < d \\ \vdash \\ n < d \vee n > 0 \end{array} \right\} \text{OR\_R1} \\ \left. \begin{array}{l} n = d \\ \vdash \\ n < d \vee n > 0 \end{array} \right\} \end{array}$$

$$\begin{array}{l} n < d \\ \vdash \\ n < d \end{array} \text{ HYP}$$